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with respect to the gas phase (infinitely dispersed) is only 12 cal/mole. These differences between the actual behaviour and that of the classical model are due to zero-point energy.

A similar situation occurs in ³He although, since the atomic mass is smaller and the interatomic potential the same, the zero-point energy has an even greater influence here. In addition to the mass difference, the nucleus of ³He has a spin value of $\frac{1}{2}$ while that of ⁴He has no spin. This has important consequences for the relative entropies of the liquid and solid phases of ³He at low temperatures and leads to a minimum in the melting curve of ³He at about 0.3 °K. In this paper we are concerned with the solid phase region above



FIGURE 1. (a) Phase diagram of ⁴He. (b) Phase diagram of ³He.

about 3 °K and at pressures less than about 2000 atm. In this region, it appears that the nuclear spins are randomly oriented so that there is an additional entropy of $R \ln 2$ which is absent in ⁴He (cf. the end of § 3·1 below). This spin difference is also associated with a difference in the statistics appropriate to the two isotopes and this shows up very remarkably in the very different properties of the two liquid isotopes. (For a review of the properties of solid and liquid ³He see, for example, Bernardes & Brewer 1962.)

In both the solid isotopes of helium there exist at least three different structures, bodycentred cubic, hexagonal close-packed and face-centred cubic. The phase diagram for both solid isotopes is shown in figure 1. According to classical lattice theory, a solid composed of atoms which have central, additive short-range forces should have a close-packed structure. In helium where the condition of central, additive short-range forces between atoms should be realized, we might therefore expect to find either of the two close-packed structures: on the other hand, the existence of the b.c.c. structure is unexpected. Almost certainly this is another illustration of the influence of zero-point energy.

† For a review of work on solid ⁴He up to 1956 see, for example, Domb & Dugdale (1957).

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